UDC 539.3

OPTIMIZATION OF ELASTIC SHELLS OF REVOLUTION

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An algorithm based on the linearization method [1] is proposed for the numerical solution of problems of optimization of elastic shells of revolution subjected to static load. Shells with fixed parameters are calculated by the finite-difference method. Procedure for determining gradients of functionals obtained by solving equations of shell equilibrium, and of load gradients at stability loss by varying shell parameters is presented. Examples are given of optimization of the shape of shells of revolution.

1. Equations of equilibrium and stability of shells of revolution and methods of their solution. Equations of equilibrium and stability of elastic shells of revolution subjected to axisymmetric load and an algorithm for their solution based on the investigations in [2] are presented below. The self-conjugate boundary value problems of equilibrium and stability are considered.

The relations between increments of strain e_{ij}, curvature k_{ij}, and angles of rotation β_i $(i, j = 1, 2)$ of shell elements to increments of displacements u, v, and w at stability loss are assumed to be of the form

$$\beta_1 = -\frac{\partial w}{\partial s} + \frac{u}{R_1}, \quad \beta_2 = -\frac{\partial w}{a \partial \varphi} + \frac{v}{R_2} \tag{1.1}$$

$$e_{11} = \frac{\partial u}{\partial s} + \frac{w}{R_1} + \lambda \psi \beta_1, \quad e_{22} = \frac{\partial v}{a \partial \varphi} + \frac{1}{a}\frac{\partial a}{\partial s} u + \frac{w}{R_2}$$

$$2e_{12} = \frac{\partial v}{\partial s} + \frac{\partial u}{a \partial \varphi} - \frac{1}{a}\frac{\partial a}{\partial s} v + \lambda \psi \beta_2$$

$$k_{11} = \frac{\partial \beta_1}{\partial s}, \quad k_{22} = \frac{\partial \beta_2}{a \partial \varphi} + \frac{1}{a}\frac{\partial a}{\partial s}\beta_1$$

$$2k_{12} = \frac{\partial \beta_2}{\partial s} + \frac{\partial \beta_1}{a \partial \varphi} - \frac{1}{a}\frac{\partial a}{\partial s}\beta_2 + \frac{1}{2a}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)\left[\frac{\partial (av)}{\partial s} - \frac{\partial u}{\partial \varphi}\right]$$

$$R_1 = \frac{\partial s}{\partial \theta}, \quad R_2 = \frac{a}{\sin \theta}, \quad \cos\theta = \frac{\partial a}{\partial s}, \quad 0 \leqslant s \leqslant S$$

where s is the length of the meridian arc; φ is the angle of rotation of the meridian plane about the axis of revolution taken as a coordinate on the parallel; a is the distance of the shell middle surface from the axis of revolution; θ is the angle between a normal to the middle surface and the axis of revolution; R_1 and R_2 are radii of curvature; u, v, and w are increments of displacements along the meridian, the parallel, and in the direction of the outward normal to the shell middle surface, respectively; λ is the eigenvalue representing external load; ψ is the angle of rotation of a shell element about a parallel in the precritical state when $\lambda = 1$. Here, and in what follows subscripts 1 and 2 denote quantities at shell cross-sections orthogonal to the meridian and the parallel, respectively; E is Young's modulus, ν is Poisson's ratio, and h is the shell thickness.

The increments of stress and moments at stability loss satisfy the following variational equations:

$$\int_0^{2\pi}\int_0^S \Big[N_{11}\delta e_{11} + N_{22}\delta e_{22} + 2N_{12}\delta e_{12} + M_{11}\delta k_{11} + M_{22}\delta k_{22} + \tag{1.2}$$

$$2M_{12}\delta k_{12} + Q_1\delta\left(\beta_1 + \frac{\partial w}{\partial s} - \frac{u}{R_1}\right) + Q_2\delta\left(\beta_2 + \frac{\partial w}{a\partial\varphi} - \frac{v}{R_2}\right) +$$

$$\lambda(\chi_1\beta_1\delta\beta_1 + \chi_2\beta_2\delta\beta_2)\Big]ds\,d\varphi = \int_0^{2\pi}\Big[N_{11}^*\delta u + \left(N_{12} + \frac{1}{R_v}M_{12}\right)^*\delta v +$$

$$\left(Q_1 + \frac{\partial M_{12}}{a\partial\varphi}\right)^*\delta w + M_{11}^*\delta\beta_1\Big]_{s=0}^{s=S} d\varphi$$

$$R_v^{-1} = {}^3/_2 R_2^{-1} - {}^1/_2 R_1^{-1}$$

where δ is the sign of variation; N_{ij}, M_{ij}, and Q_i $(i, j = 1, 2)$ are increments of stress, moments, and shear forces, respectively, multiplied by the distance a from the shell axis of revolution; χ_1 and χ_2 are stresses in the subcritical state, when $\lambda = 1$, multiplied by a; the asterisks denote values of respective functions when $s = 0$ or $s = S$ (i.e. are either specified and equal zero or unknown).

In conformity with Hooke's law we set

$$e_{11} = A(N_{11} - \nu N_{22}),\ e_{22} = A(N_{22} - \nu N_{11}),\quad e_{12} = A(1+\nu)N_{12}, \tag{1.3}$$
$$A = (aEh)^{-1}$$

$$k_{11} = B(M_{11} - \nu M_{22}), k_{22} = B(M_{22} - \nu M_{11}),\quad k_{12} = B(1+\nu)M_{12},$$
$$B = 12(aEh^3)^{-1}$$

Let us formulate conditions at the closed top of the shell of revolution, where $s = a = 0$, in the absence of concentrated forces there. In that case we must delete in the right-hand side of the variational equation (1.2) the integral of φ when $s = 0$.

Carrying out the separation of variables we substitute into Eqs. (1.1) – (1.3) the expansions of functions u, w, β_1, e_{11}, e_{22}, k_{11}, k_{22}, N_{11}, N_{22}, M_{11}, M_{22}, and Q_1 and v, β_2, e_{12}, k_{12}, N_{12}, M_{12}, and Q_2 in Fourier series in $\cos m\varphi$ and $\sin m\varphi$, respectively, $(m = 0, 1, 2, \ldots)$.

It follows from (1.2) that

$$\lim_{s\to 0}\sum_{m=0}^{\infty}[N_{11,m}\delta u_m + (N_{12,m} + R_v^{-1}M_{12,m})\delta v_m + (Q_{1,m} + \tag{1.4}$$
$$ma^{-1}M_{12,m})\delta w_m + M_{11,m}\delta\beta_{1,m}] = 0$$

where the subscript m denotes coefficients at $\cos m\varphi$ and $\sin m\varphi$ in Fourier series of respective functions.

The variations δu_m, δv_m, δw_m, and $\delta\beta_{1,m}$ are not independent. At any arbitrary point of the shell the following relationships are satisfied:

$$u = -\zeta\sin\theta + (\xi\cos\varphi + \eta\sin\varphi)\cos\theta,\ v = \eta\cos\varphi - \xi\sin\varphi \tag{1.5}$$
$$w = \zeta\cos\theta + (\xi\cos\varphi + \eta\sin\varphi)\sin\theta$$

where ξ, η, and ζ are increments of displacements of the shell middle surface at stability loss in the directions $\varphi = 0$ and $\varphi = \pi / 2$ in the plane orthogonal to the shell axis of revolution and along that axis, respectively. Passing in (1.5) to limit with $a \to 0$ and $s \to 0$ we obtain owing to the continuity of displacements the same formulas for the shell top, where $s = 0$ and ξ, η, and ζ are independent of angle φ.

Let us further assume that the angle between opposite meridians at the shell top remain unaltered in the course of deformation. Then the angle of rotation of a shell element $\beta_1(s, \varphi)$ about the parallel satisfies at the limit $s \to 0$ the condition $\beta_1(0, \varphi) = -\beta_1(0, \varphi + \pi)$ and, consequently, its expansion in Fourier series contains only odd terms

$$\beta_1(0, \varphi) = \sum_{m=1,3,5,\ldots} (\beta_{1,m} \cos m\varphi + \beta'_{1,m} \sin m\varphi) \tag{1.6}$$

When $s = 0$, owing to the arbitrariness of variations of ξ, ζ, and $\beta_{1,m}$ $(m = 1, 3, 5, \ldots)$, from (1.4) $-$(1.6) we have the relationships

$$\begin{aligned}
&u_0 \cos\theta + w_0 \sin\theta = 0, \quad Q_{1,0}\cos\theta - N_{11,0}\sin\theta = 0, \ \beta_{1,0} = 0 \ (m=0)\\
&u_1 + v_1\cos\theta = 0, \ N_{11,1}\cos\theta - N_{12,1} + Q_{1,1}\sin\theta + \frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \times \\
&\qquad M_{12,1} = 0\\
&w_1 + v_1 \sin\theta = 0, \quad M_{11,1} = 0 \quad (m = 1)\\
&u_m = v_m = w_m = 0 \quad (m = 2, 3, 4, 5, \ldots)\\
&\beta_{1,m} = 0 \quad (m = 2, 4, 6, \ldots), \ M_{11,m} = 0 \quad (m = 3, 5, 7, \ldots)
\end{aligned} \tag{1.7}$$

which provide boundary conditions for the equations of stability with separated variables when $s = 0$.

Note that when $m = 1$, the second of conditions (1.7) is also implied by that the principal vector of forces acting at the boundary of the small circular neighborhood of the shell top projected on a plane orthogonal to the axis of revolution is equal zero, namely by the equality

$$\int_0^{2\pi} \left\{\left[\left(Q_1 + \frac{\partial M_{12}}{a\partial\varphi}\right)\sin\theta + N_{11}\cos\theta\right]\cos\varphi - \left(N_{12} + \frac{1}{R_v}M_{12}\right)\sin\varphi\right\}d\varphi =$$
$$\pi\left[Q_{1,1}\sin\theta + N_{11,1}\cos\theta - N_{12,1} + \frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)M_{12,1}\right] = 0$$

At the smooth (not conical) top of the shell of revolution $\theta = 0$, $R_1 = R_2$ when $s = 0$, and conditions (1.7) assume the form

$$\begin{aligned}
&u_0 = 0, \quad Q_{1,0} = 0, \quad \beta_{1,0} = 0 \quad (m = 0)\\
&u_1 + v_1 = 0, \quad N_{11,1} - N_{12,1} = 0, \quad w_1 = 0, \quad M_{11,1} = 0 \quad (m = 1)
\end{aligned} \tag{1.8}$$

which for $m = 2, 3, 4, \ldots$ remains unchanged.

Equations for the precritical state are derived from Eqs. (1.1) $-$(1.8) by cancelling in them terms that contain the eigenvalues λ, and by taking u, v, w, β_i, e_{ij}, k_{ij}, N_{ij}, M_{ij}, and Q_i $(i, j = 1, 2)$ as functions in the precritical state and not as their increments, and adding to the right-hand side of Eq. (1.2) the work of external forces in the variations of displacements

$$\int_0^{2\pi}\int_0^S (p_1\delta u + p_2\delta v + p_3\delta w)\, a\, ds\, d\varphi$$

where p_1, p_2, and p_3 are components of external load. In the axisymmetric state $v = \beta_2 = e_{12} = k_{12} = N_{12} = M_{12} = Q_2 = p_2 = 0$.

The equations of the precritical state and the equation of stability of the shell after the separation of variables are approximated on a nonuniform network of nodes using the finite-difference equations given in [2].

The critical (bifurcation) loads at loss of stability for various numbers $m = 0, 1, 2, \ldots$ of waves on a parallel are determined as the lowest, in absolute value, eigenvalues λ of the finite-difference equations of stability obtained by the method of iterations [3] for respective numbers m.

If eigenvalues of different signs are present and the calculated eigenvalue corresponds to a load in opposite direction (of opposite sign) to be considered one, the eigenvalue of required sign may be obtained by a shift of the eigenvalue spectrum [3]. In the numerical examples presented below the eigenvalues of the lowest absolute value, determined by the method of iterations, related to a load of specified direction, and it was not necessary to shift eigenvalues.

Since the equations of the precritical state and those whose solutions are used for determining eigenvectors in the iteration process differ from equations that correspond to the last nodes of finite-difference approximations only by coefficients, numbers m, and vectors in their right-hand sides, hence they are solved jointly using the same computer program. They are transformed in three-point finite-difference equations (each equation contains unknown functions only in three adjacent nodes) in four network functions u, v, w, and M_{11} with symmetric matrices of coefficients and are then solved by the method of matrix runs [4].

2. Determination of stress intensity gradients and of load at stability loss by varying shell parameters. Stress intensity at any arbitrary point of the shell has the form of functional $\Phi = \Phi(c, X)$ of the finite-dimensional vector of the shell variable parameters $c = (c_1, c_2, \ldots, c_k)$ and of solution X of the finite-difference equations of the precritical state, which are of the form [2]

$$A_0 X = f \tag{2.1}$$

where A_0 is a symmetric matrix of coefficients; f is the load vector; X is the vector of network functions of displacements, stresses, and moments determined at nodes of the network of the finite-difference approximation. Matrix A_0, vector f, and consequently, also vector X depend on the shell variable parameters c.

Gradient $\Phi(c, X)$ is determined in terms of c as follows. By varying $\Phi(c, X)$, we obtain

$$\delta\Phi = \Phi_c^T \delta c + \Phi_X^T \delta X$$

where Φ_c and Φ_X are vectors of partial derivatives of Φ with respect to components of vectors c and X, respectively, and the superscript T denotes the transposition of a column vector to a row vector.

Solving the system of equations

$$A_0 Y = \Phi_X \tag{2.2}$$

we obtain

$$\delta\Phi = \Phi_c{}^T \delta c + Y^T A_0 \delta X = \Phi_c{}^T \delta c + Y^T (\delta f - \delta A_0 X) = \delta c^T \nabla \Phi$$

Variations δf and δA_0 are expressed in explicit form in terms of δc, and the gradient $\nabla\Phi$ of functional Φ is defined in terms of c as the vector of coefficients of components of vector δc.

Note that systems (2.1) and (2.2) differ only by the vectors in their right-hand sides and are, consequently, solved jointly on the same computer program.

The load at stability loss is determined as the smallest eigenvalue λ of the finite-difference stability equations of the form [2]

$$AU = \lambda BU \tag{2.3}$$

where A and B are symmetric matrices of coefficients calculated for m waves of the shell shape along the parallel at stability loss for which λ is minimum (matrix A_0 in (2.1) is the same as A when $m = 0$, and U is the vector of Fourier series at $\cos m\varphi$ and $\sin m\varphi$ of increments of displacements, stresses and moments at nodes of the finite-difference approximation at stability loss.

The variation of λ is determined as the variation of the eigenvalue to which corresponds the unique eigenvector U by formula

$$\delta\lambda = U^T (\delta A - \lambda\delta B) U / (U^T B U) \tag{2.4}$$

Number m is not varied, since its effect on λ is minimal.

Matrix B depends on stresses χ_1 and χ_2, and on the shell angle of rotation ψ in the precritical state, which are all functions of parameters $c_1, c_2, \ldots, c_k$. Variations $\delta\chi_1, \delta\chi_2, \delta\psi$ appear in (2.4) in the form of the scalar product $\Psi_X{}^T \delta X$, where Ψ_X is the vector of partial derivatives of the Rayleigh ratio $\Psi = U^T A U / (U^T B U)$ with respect to components of the vector of functions of the shell precritical state at nodes of the finite-difference approximation X.

After solving the system of equations

$$A_0 Z = \Psi_x \tag{2.5}$$

we carry out the transformation

$$\Psi_X{}^T \delta X = Z^T A_0 \delta X = Z^T (\delta f - \delta A_0 X)$$

and obtain $\delta\lambda = \delta c^T \nabla\lambda$ which is used for determining the load gradient $\nabla\lambda$ in terms of parameters $c_1, c_2, \ldots, c_k$ at stability loss as the vector of coefficients at components of vector δc.

The determination of stress intensity gradients and of load at stability loss thus requires the solution of problems (2.2) and (2.5) for the finite-difference equations of the shell precritical state.

3. The optimization algorithm. We use the linearization method [1] for shell optimization, which entails successive variation of parameters $c_1, c_2, \ldots, c_k$, which may specify the shape of the shell middle surface, its thickness, modulus of elasticity, etc., whose input values are arbitrary.

We shall describe the algorithm for determining a shell whose material volume V is minimum, with the bifurcation load p_* defined as the lowest eigenvalue of the finite-difference stability equations, is not lower than the specified p_B $(p_* \geqslant p_B)$, and the intensity of stresses σ_1 and σ_2 at the precritical state along the meridian and the parallel, respectively, under load $p \leqslant p_B$ does not exceed the admissible stress σ_B, i.e. $\sigma_* \leqslant \sigma_B$, where $\sigma_* = \max \sigma$, $0 \leqslant s \leqslant S$, and σ is the maximum intensity of stresses $(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)^{1/2}$ across the shell thickness for fixed s.

Direction of the vector of increments of parameters $c_1, c_2, \ldots, c_k$ at any single step of the optimization process is determined by vector b obtained from the solution of the following problem. Determine the maximum of $(b^T\nabla f_0(c) + {}^1/_2 b^T b)$ under the condition that

$$f_i(c) + b^T\nabla f_i(c) \leqslant 0, \quad \text{if} \quad f_i(c) > \max(-\varepsilon, f_1(c) - \varepsilon, f_2(c) - \varepsilon) \quad (i = 1, 2) \tag{3.1}$$

$$\varepsilon > 0, \quad f_0(c) = \frac{V(c)}{k_V}, \quad f_1(c) = 1 - \frac{p_*(c)}{p_B}, \quad f_2(c) = \frac{\sigma_*(c)}{\sigma_B} - 1$$

where c is the considered point of values of $c_1, c_2, \ldots, c_k$; the superscript T denotes the transposition of a column vector to a row vector; $k_V = \text{const}$ is the factor of transition to dimensionless quantities, and σ_* is calculated for $p = p_B$.

The vector of increments of parameters $c_1, c_2, \ldots, c_k$ is assumed to be $\Delta c = b / 2^i$, where i is the first of numbers $0, 1, 2, \ldots$ for which the inequality

$$f_0(c + \Delta c) + \Lambda F(c + \Delta c) < f_0(c) + \Lambda F(c) - 2^{-i}(b^T b)^{1/2}\varepsilon'$$

$$F(c) = \max(0, f_1(c), f_2(c)), \quad 0 < \varepsilon' < 1$$

where the number $\Lambda > 0$ must be greater than the sum of Lagrange multipliers of problem (3.1), is satisfied. Numbers k_V, ε, and ε' determine the admissible length of vector Δc. In numerical examples presented below these numbers were selected so that vector b can be taken as vector Δc, which considerably reduces the volume of calculations. From the selected point c we pass to point $c + \Delta c$ which is then taken as the input point for the next optimization step.

Note that σ_* and p_* are not continuously differentiable functions of $c_1, c_2, \ldots, c_k$. If σ attains its maximum value at more than one point of the shell, and the variation of parameters c in the direction determined by the gradient of σ at only one point results in an inadmissible increase of σ at other points of the shell where it was close to σ_*, then it is necessary to impose in problem (3.1) constraints on the increments of σ at such points. Similarly, in the case of bunching of the spectrum of eigenvalues of shell stability equations around the eigenvalue of p_* it may be necessary to impose constraints on the increments on some eigenvalues closest to that of p_*, and determine the variation of p_*, taking into account all of its corresponding eigenvectors, if there are more than one of these. In the examples considered below such refinement of the algorithm was not necessary.

In the problem of optimization of a shell subjected to inertia forces induced by uniformly accelerated motion (or, what is equivalent, by its own weight) we find $\max_c \min(g_*, g_\sigma)$, where g_* is the lowest acceleration of motion at which loss of shell stability takes place, and g_σ is the acceleration at which the maximum intensity of stresses σ_* equals the specified σ_B. Owing the linearity of equations of the pre-critical state, σ_* varies in proportion to acceleration, hence $g_\sigma = g\sigma_B / \sigma_*$, where σ_* is calculated for an acceleration equal g.

Using the algorithm of minimax determination [1], we determine the direction of an optimization step by vector b for which the quantity $(\mu + {}^1/_2 b^T b)$ is minimal with respect to μ and b under the condition that

$$f_i(c) + b^T \nabla f_i(c) + \mu \geqslant 0, \quad \text{if} \quad f_i(c) < F(c) + \varepsilon \quad (i = 1, 2);\ \varepsilon > 0 \quad (3.2)$$

$$(f_1(c) = g_*(c) / k_g,\ f_2(c) = g_\sigma(c) / k_g,\ F(c) = \min(f_1(c), f_2(c)))$$

where $k_g = \text{const}$ is the coefficient of transition to dimensionless quantities. Gradient g_* of parameters $c_1, c_2, \ldots, c_k$, as well as the gradient of p_* in problem (3.1), is calculated as the gradient of the simple (not multiple) eigenvalue of the finite-difference equations of stability.

The vector of increments of parameters $c_1, c_2, \ldots, c_k$ is assumed to be $\Delta c = b / 2^i$, where i is the first of numbers $0, 1, 2, \ldots$ for which the inequality

$$F(c + \Delta c) < F(c) - 2^{-i} (b^T b)^{1/2} \varepsilon', \quad 0 < \varepsilon' < 1$$

is satisfied. From point c we pass to point $c + \Delta c$, and so on.

Vector b in problems (3.1) and (3.2) is obtained by testing all possible solutions by the Kuhn — Tucker condition.

4. Results of calculations. The following numerical examples illustrate the optimization of the shape of elastic shells of revolution of constant thickness, whose generatrices are defined by functions of the form

$$\frac{z}{a_N} = \frac{\cos t\gamma - \cos\gamma}{\sin\gamma} + \sum_{i=0}^{4} c_i (1 - t^{2i+2})$$

$$\frac{a}{a_N} = \frac{\sin t\gamma}{\sin\gamma} + \sum_{i=1}^{4} c_{i+4} t (1 - t^{2i}), \quad 0 \leqslant t \leqslant 1$$

where a is the distance of the shell middle surface from the axis of revolution; z is the distance measured along the axis of revolution; at the shell top $t = 0$ and $a = 0$, while at the rim $t = 1$, $z = 0$, and $a = a_N$; coefficients $\gamma, c_0, c_1, \ldots, c_8$ are independent of t. When $c_0 = c_1 = \ldots = c_8 = 0$ these functions define a spherical shell. The Poisson's ratio is assumed to be $\nu = 0.3$.

When solving equations of shell stability, the iteration process was terminated when eigenvalues λ, displacements u, v, and w, and moment M_{11} at nodes s_n ($n = 0, 1, \ldots, 50$) of the finite-difference approximation at the r-th and $(r + 1)$-st iterations satisfied inequalities of the form

$$|\lambda^{(r+1)} - \lambda^{(r)}| < 10^{-4} |\lambda^{(r+1)}|, \quad \max_{0 \leqslant n \leqslant 50} |u_n^{(r+1)} - u_n^{(r)}| < 10^{-4} \sum_{n=0}^{50} |u_n^{(r)}|$$

where the superscript denotes the iteration ordinal number and the subscript, that of the node s_n. The number of nodes of the finite-difference approximation $0 = s_0 < s_1 < \ldots < s_{50} = S$ was 50 with a step $(s_{n+1} - s_n) / S = 0.025$, $0 \leqslant n \leqslant 29$; 0.015, $30 \leqslant n \leqslant 39$; 0.010, $40 \leqslant n \leqslant 49$.

In the first example the volume of material of the shell subjected to a nonuniform hydrostatic pressure of intensity $q(1 - 0.1\, z / a_N)$ (coefficients $q = \text{const}$) was minimized on the assumption that $q_* \geqslant q_B$ and $\sigma_* \leqslant \sigma_B$ for $q = q_B$ where q_* is the value of q at loss of the shell stability, σ_* is the maximum stress intensity in the precritical state, and σ_B and q_B are specified quantities.

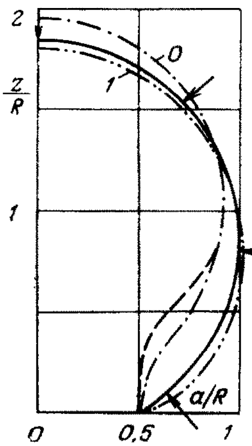

Fig. 1

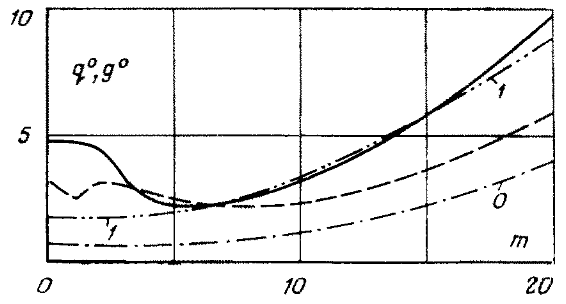

Fig. 2

Conditions of hinged support with all displacement components and bending moments equal zero were assumed to apply at the rim.

The optimization process was first applied to a spherical shell of diameter $R = 2a_N$ and thickness $h = h_0 = a_N / 10$ with $\gamma = 150°$, $c_0 = c_1 = \ldots = c_8 = 0$. For such shell $q_* = q_B = 0.002225\, E$, $\sigma_* = 14.94\, q$, $\sigma_B = \sigma_* = 0.0333\, E$ when $q = q_B$. Its generatrix is shown in Fig. 1 prior to deformation by the solid line circle segment, in the precritical state by the dash-dot curve 0, and by the dash line at stability loss with $m = 6$ in the meridional cross section where displacement in the circumferential direction is $v = 0$ and a dent appears in the shell. In Figs. 1 and 2 load vectors are indicated by arrows.

Let us now vary the shell thickness h and parameters $c_0, c_1, \ldots, c_8$ maintaining the internal volume and radius a_N of the shell, and angle γ constant. As the result of optimization for nearly constant σ_* and q_*, the volume of shell material was reduced by approximately 15% for $h = 0.8481\, h_0$, $c_0 = 0.09616$, $c_1 = -0.02188$, $c_2 = -0.04606$, $c_3 = -0.06885$, $c_4 = -0.08909$, $c_5 = -0.002743$, $c_6 = 0.001229$, $c_7 = 0.01123$, $c_8 = 0.02393$. The shape of the last shell is shown in Fig. 1 by the dash-dot curve 1. The spectrum of critical values of q in terms of the number $m = 0, 1, 2, \ldots$ of waves of the shape along the parallel at stability loss is shown in Fig. 2 for the initial spherical and the last optimal shells by solid and dashed lines, respectively ($q^0 = 10^3 q / E$).

The optimal shape of a shell of revolution with a fixed rim ($u = v = w = \beta_1 = 0$ and $s = S$) subjected to inertia forces induced by uniform acceleration along the axis of revolution z is determined in the second example. The shell thickness h and distance a_N are fixed, and $h / a_N = 0.02$. Unlike in the first example, the internal shell volume is not fixed. The quantities $\gamma, c_0, c_1, \ldots, c_8$ are varied.

An initially spherical shell is considered for which $\gamma = 30°$, $c_0 = c_1 = \ldots = c_8 = 0$, and $z(0) = (2 - \sqrt{3})\, a_N \approx 0.268\, a_N$, the volume of material $V = 1.072\ \pi h a_N^2$, the maximum stress intensity in the precritical state $\sigma_* = 1.735 \rho a_N g$ (ρ is the unit volume mass of the material and g is the acceleration of the shell motion), and the lowest g at which stability loss occurs in $g_* = 0.006141\ E / (\rho a_N)$. It is assumed that the maximum stress intensity σ_B of this shell occurs when $g = g_\sigma = g_*$, hence $\sigma_B = 1.735\ \rho g_* a_N = 0.01065\ E$.

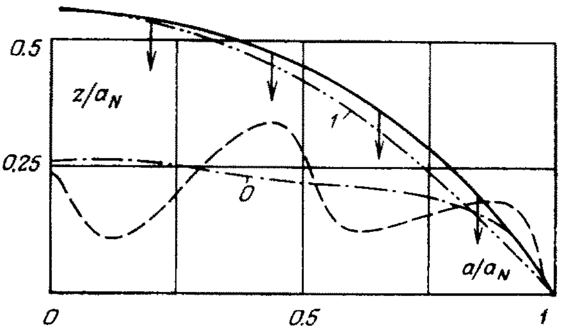

Fig. 3

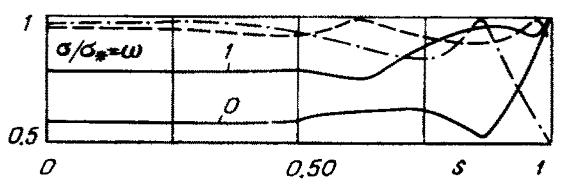

Fig. 4

The shell whose generatrix is shown in Fig. 3 by the solid curve was determined with the use of the optimization algorithm described in Sect. 3. For this shell $\gamma = 32.7°$, $c_0 = 0.09529$, $c_1 = 0.03703$, $c_2 = 0.03479$, $c_3 = 0.04321$, $c_4 = 0.05348$, $c_5 = -0.03405$, $c_6 = -0.03204$, $c_7\ -0.02330$, $c_8 = -0.01369$. The volume of its material is by approximately 20% greater than that of the initial spherical shell. The maximum stress intensity $\sigma_* = 0.7505\ \rho g a_N$ and the lowest acceleration at which stability loss occurs $g_* = 0.01685\ E / (\rho a_N)$. Stress σ_* reaches the maximum admissible value σ_B, before loss of shell stability takes place, at acceleration $g_\sigma = 0.0142\ E / (\rho a_N)$ which is more than twice higher than the acceleration $g = 0.006141\ E / (\rho a_N)$ admissible for the initial spherical shell.

Shapes of the optimal shell generatrix are shown in Fig. 3 by the dash-dot curve O for the precritical state and by the dash curve at stability loss ($m = 1$). The spectrum of critical accelerations g that correspond to stability loss with $m = 0, 1, 2, \ldots$ waves along the parallel is shown in Fig. 2 by curve 1 ($g^0 = 10^2\, g \rho a_N / E$), while the lower curve O defines that spectrum for the initial spherical shell.

The maximum stress intensity σ as a function of the shell thickness s for the initial spherical and the optimized shells in the subcritical state is shown in Fig. 4 by curves O and 1, respectively. Optimization obviously leads to a smoothing of distribution of σ along the shell length.

The stressed state of the optimal shell is close to the zero-moment state. In Fig. 4

the dash-dot and the dash lines show the ratio ω of stress intensity at the shell middle surface to its maximum value across the shell thickness as a function of s for the initial spherical and the optimal shells, respectively.

It should be noted that in these examples the obtained shells are sensitive to deviations from their shape. Thus in the last example the admissible acceleration $g = g_\sigma = 0.00717\ E\ /\ (\rho a_N)$ for the shell obtained by an additional variation of the optimum shell parameters, shown in Fig. 3 by the dash-dot curve 1, is half of that for the shell taken as optimal shown in that figure by the solid line. In that case loss of stability takes place at acceleration $g_* = 0.0105\ E\ /\ (\rho a_N)$. Hence the possibility of actual reduction of the maximum stress intensity and of stability improvement of shells considered above by varying their shape is insignificant.

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